3/13/20 More Properties of Logs and the Natural Logarithm

Recall that a logarithm without a base is the Common Logarithm. It has a base of 10. These logarithms can be calculated with a calculator.

Evaluate the following logarithms:

Ex 1: $\log 73$

$= \log_{10} 73$

Calculator!

$\log(73) \approx 1.863$

Ex 2: $\log 1026$

$\log(1026)$

≈ 3.011

Ex 3: $\log 0.9$

$\log(0.9)$

≈ -0.0458

Ex 4: $\log 0.0087$

$\log(0.0087)$

≈ -2.0604

What if I want to calculate a logarithm that does NOT have base 10? Well, we are in luck! We have a formula we can use to rewrite the logarithm so that we can use our calculator!! This formula will change the base of the logarithm so that we can rewrite the logarithm as a base of 10.

Change of Base Formula:

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$$\log_a b = \frac{\log(b)}{\log(a)}$$

Remember that $\log(b) = \log_{10} b$

called the common log

Evaluate the following:

Ex 5: $\log_5 625$

$= \dfrac{\log 625}{\log 5}$

Plug in!

$= 4$

Ex 6: $\log_8 2.7$

$= \dfrac{\log 2.7}{\log 8}$

≈ 0.478

Ex 7: $\log_7 55$

$= \dfrac{\log 55}{\log 7}$

≈ 2.059

Ex 8: $\log_2 0.03$

$= \dfrac{\log 0.03}{\log 2}$

≈ -5.059

The history of mathematics is marked by the discovery of special numbers such as counting numbers, zero, negative numbers, π, and imaginary numbers. In this lesson you will study, e, one of the most famous numbers of modern times. Like π and i, the number e, is denoted by a letter. The number is called the natural base e, or the Euler number after its discoverer, Leonhard Euler (1707 – 1783).

NATURAL LOGARITHM:
The logarithm with base e is called the natural logarithm, so we can use $\log_e$, but it is more often denoted by ln.

NATURAL LOG: $\log_e x = \ln x$

Natural base *e*: e is like π. It is irrational. $e \approx 2.71828...$

* e is a number, not a variable.

The properties of logarithms that we have studied also apply to the natural logarithm.

LOG PROPERTIES REVISITED. FOR NATURAL LOGS:

1. $\ln e = 1$
2. $\ln 1 = 0$
3. $\ln(xy) = \ln x + \ln y$
4. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$
5. $\ln x^n = n \cdot \ln x$

Let's use these properties to work with natural logarithms.

Simplify:

Ex 9: $\ln e^5$

$= 5$

Ex 10: $\ln\left(\frac{1}{e^{24}}\right)$

$= \ln e^{-24}$

$= -24$

Ex 11: $\ln e^{-3} + \ln e^{12}$

$-3 + 12$

$= 9$

Ex 12: $\frac{\left(\ln e^2\right)^6}{\left(\ln e^3\right)^4}$

$= \frac{\ln e^{12}}{\ln e^{12}} = \boxed{1}$

Expand or condense the expression:

Ex 13: $\ln(6^3 \cdot 8^4)$

$= \ln 6^3 + \ln 8^4$

$= 3\ln 6 + 4\ln 8$

Ex 14: $3\ln 6 - 4\ln x$

$= \ln 6^3 - \ln x^4$

$= \ln \frac{6^3}{x^4}$

Ex 15: $\ln 16 - \ln 4$

$= \ln \frac{16}{4}$

$= \ln 2$

Ex 16: $3(\ln 3 - \ln x) + (\ln x - \ln 9)$

$= \ln \frac{3^3}{x^3} + \ln \frac{x}{9}$

$= \ln \frac{9x}{9x^{3 \cdot 2}} = \ln \frac{1}{x^2}$

Ex 17: $\ln 20 + 2\ln\frac{1}{2} + \ln x$

$= \ln 20 + \ln\left(\frac{1}{2}\right)^2 + \ln x$

$= \ln\left(20 \cdot \frac{1}{4} \cdot x\right)$

$= \ln 5x$

Ex 18: $\ln 4x^2 y$

$\ln 4 + \ln x^2 + \ln y$

$= \ln 4 + 2\ln x + \ln y$